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THE ELECTRIC STRENGTH OF AIR AT ATMOSPHERIC PRESSURE UNDER ALTERNATING AND CONTINUOUS POTENTIALS

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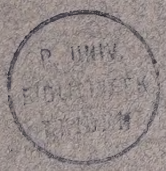

By
WILLIAM S. BROWN

June, 1916

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THE ELECTRIC STRENGTH OF AIR.—VII

BY J. B. WHITEHEAD AND W. S. BROWN

ABSTRACT OF PAPER

In view of the variation among the values obtained by different observers, this paper aims to make a careful determination of corona-forming voltages for alternating and for positive and negative continuous voltages in the same apparatus and under the same conditions.

Within the range of wire diameters used, corona appears at a lower value when the wire is positive than when it is negative, although the two curves converge for increasing diameters. The maximum excess of negative over positive, as observed, was 6.3 per cent.

The values with alternating voltage coincide with those of negative continuous voltage. Positive continuous voltage therefore forms corona at the lowest value.

The observations on the negative corona give values higher than any heretofore obtained.

Other experiments are described, giving qualitative indications of the correctness of Townsend's theory of ionization by collision.

THE voltage at which corona appears on a round wire in air depends on the diameter of the wire, the potential gradient at the surface and the density of the air. The relation as based on the results of these papers and those of Peek[1] is of the form,

$$E = A\,\delta + B\sqrt{\frac{\delta}{d}} \qquad (1)$$

in which E is the potential gradient at the surface of the wire of radius r or diameter d, δ is the so-called "density factor", its value being unity at a pressure of 76 cm. and a temperature of 25 deg. cent., and A and B are constants. In the experiments on which formula (**1**) is based alternating voltages were used, and it was shown that the maximum value of voltage or gradient determines the appearance of corona.

If the value of E as obtained from the empirical formulas of Whitehead[2] and Peek and from the results of some of the other investigators of the alternating-current corona be plotted as a function of the radius of the wire, we find that the value of E for

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any one wire shows noticeable discrepancies, *i.e.*, the constants A and B have different values. The differences in the observed values are probably due to the difficulty in measuring the ratio of maximum to effective values of the high voltage, to the unequal surface condition of the wires and to failure to properly correct for atmospheric conditions. The observations of Whitehead gave $A = 32$ and $B = 13.4$. The work of Peek[1], somewhat later, showed a similar relation between E and d, giving $A = 29.8$ and $B = 12.7$. These values in his subsequent papers have been brought up to 31.0 and 13.5 respectively, which are in close agreement with the result of Whitehead.

The most important work on the direct-current corona has been done by Watson[3], Shaffers[4], and Farwell[5]. Watson made observations with wires ranging from 0.70 mm. to 12.76 mm. in diameter, using as a source of power an influence machine of special design. For the case of a wire and cylinder he found that the polarity of the wire had a marked influence on the appearance of corona and on the value of the critical intensity. Schaffers, using wires from 0.0006 to 0.70 cm. in diameter in tubes of various sizes, found that for the larger wires the positive corona appeared at a lower voltage than the negative, while for the smaller sizes the reverse was true. The curves of critical intensity crossed at a point corresponding to a wire 0.01 cm. in radius.

Farwell, using a series of 500-volt generators as a source of high potential, investigated the influence of the polarity of the wire, of temperature, pressure and humidity on the corona, forming voltages on copper wires ranging from 0.009 to 0.258 cm. in diameter in a tube 4.45 cm. in diameter. For a given size wire, he found that corona appeared at a much lower voltage when the wire was positive. His observations can be represented by formula (**1**) and the value of the constants as given in his A. I. E. E. paper are for the positive wire, $A = 31.6$ $B = 8.47$, for the negative wire, $A = 38.0$, $B = 8.06$. Here again, as in the alternating-current case, the results of different investigators show considerable divergence.

It was in view of these differences among the values of the constants of formula (**1**) as determined by different observers that the present investigation was undertaken. The different conditions under which the several investigators worked should account in a large measure for the different values obtained for both alternating and continuous voltage. The aim of the present

experiments has been to compare alternating and continuous values of corona voltage under identical conditions and under the best possible conditions for accuracy.

Some later experiments outlined in Section V had as their purpose the testing of some of the assumptions of the present day theories as to the start and ultimate nature of corona.

Since the experiments of this paper were completed Peek[10] has published results of a comparison of alternating and continuous corona voltages. He finds that they are identical and independent of the polarity of the wire in the case of continuous voltage. The results described in this paper are noticeably at variance with those of Peek.

II. Description of Apparatus

For Alternating Voltages. The diagram of connections for the production of and measurements on the alternating-current corona is given in Fig. 1. The motor-generator G supplies the

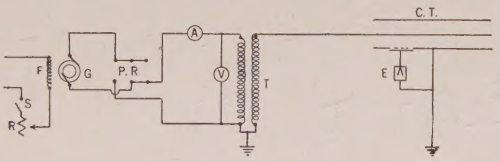


Fig. 1

low-tension winding of the transformer T through a potential regulator, $P\ R$. One terminal of the high-tension winding is connected to the low-tension winding and to ground and the other to the corona wire. $C\ T$ is the corona tube and E, an electroscope, both described in detail in a subsequent paragraph.

The motor-generator set consisted of two 5-kw., 1200-rev. per min., 120-volt alternating-current generators, one 60 and the other 20 cycles driven by a 7.5-h.p., 120-volt direct-current, shunt-wound motor. The use of a storage battery of large capacity as a source of energy for driving the motor and for exciting the two alternators made it possible to obtain a very constant voltage. Throughout the experiments only the 60-cycle unit was used. The armature was surface wound and the wave smooth.

The transformer used was rated at 3000 watts, 60 cycles, 100-25,000 volts. The low-tension winding was in two 50-volt sections, the high-tension in four 6250-volt sections. The ratio

of turns of the transformer as furnished by the manufacturer was 1-250.18 and this ratio was used to determine the secondary voltage. The primary voltage was read on a standardized Weston dynamometer voltmeter V (Fig. 1) and was controlled by the potential regulator P R and a variable resistance R in the generator field F. The influence of the varying I R drop in the primary winding was practically negligible in its effect on the ratio of transformation owing to the small values of the magnetizing and load currents.

The corona apparatus consisted of a wire stretched co-axially in a metal tube. The wire was clamped at each end in a device which permitted stretching and centering. The tube was mounted on an insulated stand which also supported a very sensitive electroscope. Small holes were drilled in a section of the tube and a disk of the same curvature as the tube was connected to one terminal of the electroscope and placed as close to the tube as possible without actual contact. The disk, the con-

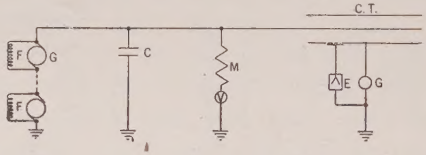


FIG. 2—CONNECTIONS FOR EXPERIMENTS WITH 15,000-VOLT DIRECT CURRENT GENERATOR

nections to the electroscope and the electroscope proper were protected by grounded metal shields. The electroscope was charged to a potential of 440 volts from the lighting mains through lamps and condensers by parallel-series connection. The insulated stand was grounded for the alternating-current observations thus grounding the other terminal of the electroscope. The ionization accompanying the very first appearance of corona causes the electroscope to discharge rapidly, which thus served as detector of corona. Fig. 4 is an illustration of the tube as used in some later experiments.

For Continuous Voltages. Two sources of high continuous voltage were available, a high-voltage generator and a kenotron or hot-cathode rectifier. The diagram of connections in the use of the former is shown in Fig. 2. G the generator, supplies the high potential through parallel condensers C to the corona wire. V M and G' are a voltmeter, multiplier and galvanometer respectively.

The generator of special design consisted of eight units mounted on an insulating bed. Each armature is provided with two windings and commutators and rated as follows: Amperes 0.1; speed, 2700 rev per min.; volts per commutator, 937.5; shunt wound. The available potential was, therefore, 15,000 volts, when the machine was run at normal speed with all the units in series.

The connection of a large number of low-voltage generators in series as a method of obtaining high values of continuous potential has been often used. In the present case to avoid insulation troubles, self-excited, shunt-wound generators were used with individual rheostats mounted on each frame and special precautions were taken in the design of the insulation. The generators were driven by a 5-h.p., 220-volt direct-current, shunt-wound motor through an insulating belt and insulated couplings. Voltage control was obtained in two ways. Adjustable carbon rod resistances connected in series with the generator fields and carefully insulated from the frame were mounted on the top of each machine and a movable insulated gearing provided uniform variation of all the rheostats. For fine adjustment a rheostat was inserted in the driving motor field which gave voltage control by varying the motor speed. An illustration of the unit is given in Fig. 5.

The Moscicki condensers, whose function will be explained later, were rated at 20,000 volts, effective value. They are of the Leyden jar type, the dielectric being of glass; and to prevent corona from forming on the edge of the plates, the condensers were filled with oil. Two condensers were used, consisting of eight units of 0.002 microfarad each.

The voltmeter used was a Weston intrument of the permanent magnet type, with two scales (0-150 and 0-750 volts). It was carefully compared throughout its entire range with a Weston laboratory standard instrument and found correct within the limit of observation error, *i.e.*, 0.1 per cent. The resistance of the high scale winding was 82,790 ohms. As it was desired to measure potentials up to 35,000 volts, a multiplier of 4.5 megohms was necessary. Shunting the instrument by an approximately equal resistance reduced this amount by one half although it introduced the disadvantage of doubling the amount of power required. For example, at 15,000 volts this demand is 0.0065 ampere and is great enough to impose a serious limitation on some of the later experiments, using the kenotron. A number

of multipliers of the regular type supplied by the voltmeter manufacturers for increasing the range of their laboratory voltmeters were available. Their total resistance, however, was only 707,630 ohms. To obtain the remainder, six units were made up in this laboratory according to specifications of the Bureau of Standards. Each unit consisted of forty mica cards wound with manganin wire. The resistance of each card was approximately 6000 ohms. They were mounted on horizontal glass rods, which in turn were supported by hard rubber uprights, and the whole mounted on a hardwood base. The current capacity was 0.022 amperes. The resistance of each unit was measured with a standard bridge and also by placing it in series with a standard Weston voltmeter of known resistance across mains of known potential.

The diagram of connections for the second method using a kenotron is shown in Fig. 3. G the high-frequency generator

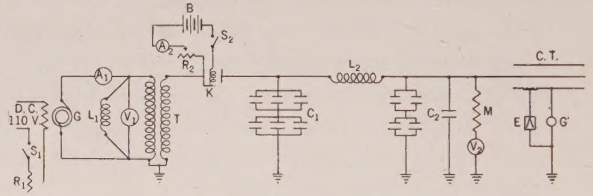


FIG. 3—CONNECTIONS FOR EXPERIMENTS WITH THE KENOTRON

supplies, through the grounded transformer T the kenotron K with high voltage. The condensers C_1 and C_2 and the inductance L_2 are introduced to smooth out the voltage which is rectified in pulsating form by the kenotron. M and V_2 are the multiplier and voltmeter, as used in the first method and $C\ T$ is the corona tube, with the electroscope E and the galvanometer G' as before. L_1 is an inductance, V_1 a hot wire voltmeter, and A_1 a hot band ammeter. Voltage control was obtained by varying the field circuit of the generator by means of the resistance R_1.

The high-frequency generator was a direct-driven unit consisting of a generator with a double field structure, one of 48 poles, the other of 240, and a 3-h.p., 240-volt direct-current, shunt-wound motor. The armature of the alternator was stationary and provided with a distributed winding. The rated full speed was 1500 rev. per min. at which one generator gave 600 cycles and the other 3000. The generators were rated at 4.4 amperes and 110 volts.

Speed or frequency control was obtained by armature and field resistance in the motor circuit, the use of the former being practically limited to starting. The driving motor and the generator field were both supplied from a storage battery which made conditions very favorable for steadiness in readings.

The kenotron used in the present investigation was designed for a maximum voltage of 40,000. It consisted of a highly evacuated glass bulb containing a cylindrical anode along whose axis was stretched a 10-mil tungsten filament, which acted as the hot cathode. The cylindrical anode was of small diameter in order to bring the anode and cathode very close together to reduce the space charge effect and thereby increase the current carrying capacity of the rectifier. One terminal was of the ordinary incandescent lamp type and served to supply energy to the cathode, acting at the same time as one terminal for the high voltage. The other end of the tube was sealed off with a metal cap and ring, which served as the other high-voltage-terminal. The theory and performance of the kenotron have been completely described by Dushman[6] and others.

For heating the tungsten filament, a 12-volt storage battery B Fig. 3 was used. A resistance R_2, a 15-ampere ammeter A_2 and a knife switch S_2 were connected in series with the battery to provide control of the filament current and indirectly the electron emission. Table I gives the electron emission and the voltage drop in the kenotron for various filament currents. A filament current of 6.1 amperes was used throughout this work.

TABLE I—CHARACTERISTICS OF THE KENOTRON

Filament current	Electron emission	Minimum voltage drop
5.0	9.5	...
5.2	16.0	86
5.4	27.0	122
5.6	40.0	158
5.8	57.0	200
6.0	82.0	250
6.1	100.0	280

The capacity C_1 (Fig. 3), was made up of six units connected two in series, three in parallel. Each unit was insulated for 10,000 volts and had a capacity of 0.01 microfarad. The con-

denser plates were immersed in oil. Spark gaps set for 10,000 volts were placed across the terminals of each unit to prevent breakdown of the insulation resulting from possible unequal distribution of potential. The capacity C_2 was made up of four similar units connected two in series, two in parallel together with the Moscicki condensers used in connection with the first method. The inductance L_1, consisted of two air coils of 360 turns of No. 10 B. & S. wire. The coils were in parallel but magnetically separated from each other. At 600 cycles they had an impedance of 9 ohms each. The high-tension winding of a 5 kw., 6600/110-volt, 60-cycle power transformer served for L_2. The function of these inductances is explained in connection with the experiments.

III. Preliminary Experiments

The approximate sizes of wire on which corona would appear for both alternating and continuous voltages with tubes of a given diameter were calculated from the results of foregoing researches. Two tubes were used, with inside diameters of 6.109 and 4.855 cm., respectively. Six sizes of wire were used, with diameters of 0.074, 0.090, 0.125, 0.166 and 0.231 cm. All were of brass except the largest, which was of steel. The wires were always carefully polished with crocus cloth and chamois before introducing them into the tube.

The first tests made were with the high-voltage generator as a source of potential. A number of runs were made on the same tube and wire and the voltage at which corona appeared, as determined by visual methods and temperature and atmospheric pressure, were recorded. An interesting source of error was developed at this point. It was found that successive runs under the same conditions gave values of corona voltage differing by several per cent. The trouble was traced by means of oscillograms to periodic fluctuations in the continuous voltage (see Fig. 6) of high frequencies which could not be associated with any feature of the rotating element.

In an effort to smooth out the inequalities of voltage, two Moscicki condensers (0.004 microfarad) were connected in parallel with the generator. The oscillograms shown in Fig. 6 show that the voltage fluctuation was increased under these conditions. Sixteen Moscicki units (0.032 microfarad) were then used and the oscillogram, Fig. 7, shows that the voltage was then practically constant. The frequency of the oscilla-

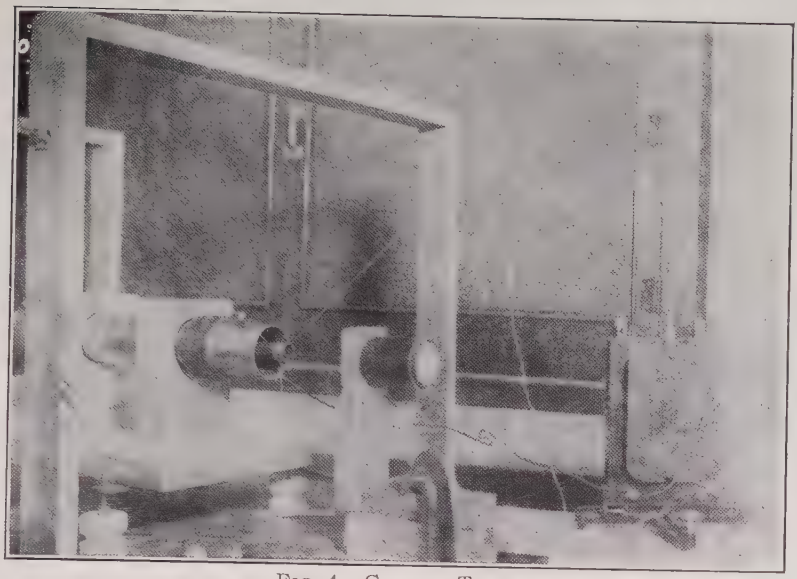
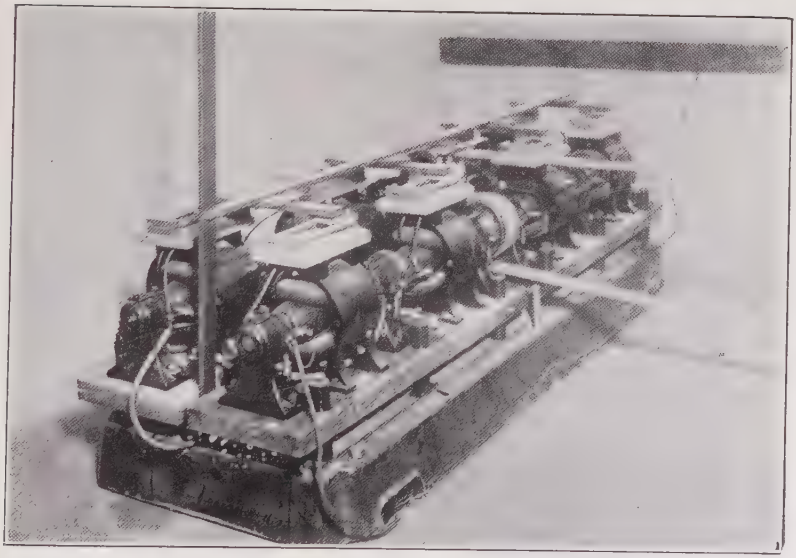


FIG. 4—CORONA TUBE



[WHITEHEAD AND BROWN]

FIG. 5—HIGH-VOLTAGE GENERATOR

FIG. 6

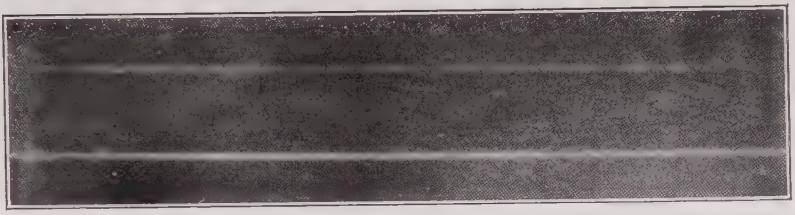


FIG. 7

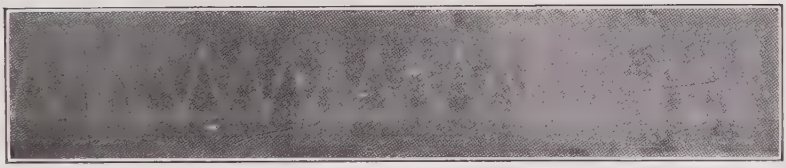


FIG. 9

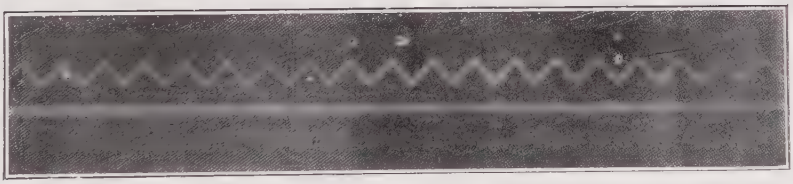
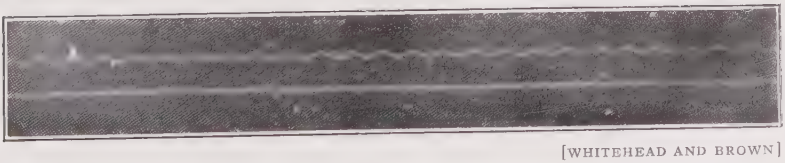


FIG. 10



[WHITEHEAD AND BROWN]

FIG. 11

tions, when the two units were used, was obtained by superimposing a wave of known frequency, not shown in Figs. 6 and 7. The frequencies as measured in this way are for the fundamental, 359.5 cycles per second, and for the superposed, 12.3 cycles per second. An effort was made to connect these frequencies with periodic recurrencies due to the speed of the motor, the generators, the belt, the number of commutator segments, etc., but without success. The fluctuation disappeared with increased capacity in parallel and is probably due to a resonance condition between the inductance of the armatures and the capacity of the circuit. The slow superimposed fluctuation could not be definitely traced. Further investigation of this interesting phenomenon had to be postponed.

A determination of E as a function of the capacity in parallel with the generator showed that above 0.016 microfarad the value of E was constant. The results are given in Table II and are plotted in the form of a curve in Fig. 8. A capacity of 0.032 microfarad was used, twice as much as actually required.

TABLE II—APPARENT LOWERING OF THE CRITICAL SURFACE INTENSITY DUE TO RESONANCE IN THE GENERATOR CIRCUIT

No. of units	Capacity m.f.	Voltmeter reading	Critical surface intensity E
1	0.002	620.0	68.6
2	0.004	555.0	61.4
3	0.006	622.5	68.9
4	0.006	647.5	71.7
5	0.010	652.5	72.2
6	0.012	662.5	73.3
7	0.014	666.0	73.7
8	0.016	665.0	73.7
9	0.018	665.0	73.7
10	0.020	665.0	73.7
11	0.022	665.0	73.7
12	0.024	665.0	73.7
13	0.026	665.0	73.7
14	0.028	665.0	73.7
15	0.030	665.0	73.7
16	0.032	665.0	73.7

It must be noted that the percentage variation in voltage as shown by the oscillogram (Fig. 9) is much greater than that actually obtaining under test conditions, as the oscillograph requires a current nearly 25 times as great as that taken by the voltmeter and corona tube and the fluctuation varies directly

with the current taken from the condensers. Under test conditions therefore—voltmeter and tube connected—the voltage variation is only a very small fraction of one per cent.

The maximum voltage obtainable from the high-voltage generator was 15,000 volts so the kenotron was used for higher values. It was soon found that the transformer used imposed a serious limitation on the value of the continuous voltage obtainable by this means. At 600 cycles the high-tension windings of the transformer took a primary charging current of 20 to 25 amperes and as the full load current of the 600-cycle generator was only 4.4 amperes, the speed and consequently the frequency rapidly dropped on connecting the transformer. Although the internal reactance of the generator was considerable, the capacity of the high-tension windings of the transformer was more than sufficient to overbalance it. Additional reactance connected

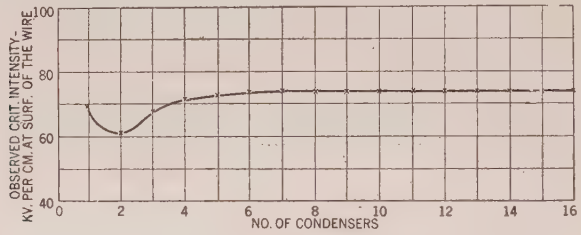


Fig. 8—Apparent Lowering of Critical Surface Intensity Due to Resonance in the Generator Circuit

accross the primary of the transformer was the obvious way of reducing this current. A number of air coils of low resistance were tried and after a number of trials the two coils as described in an earlier paragraph were selected, as they reduced the charging current to about five amperes and gave a maximum value of continuous voltage of about 26,500 volts.

The next step in the development of the kenotron method was to introduce parallel capacity in the direct-current side of the kenotron to smooth out the voltage by supplying energy to the load during the part of each cycle when the kenotron delivers no current. When operated alone *i.e.*, without condensers the rectifiers gave a continuous voltage which fluctuated between zero and a maximum with the frequency of the impressed e.m.f., 600 cycles. Four 10,000-volt condensers in series (0.0025 microfarad were connected between the direct-current side of the kenotron

and ground. The oscillogram, Fig. 10, shows the character of the continuous current voltage with the above capacity. Nine condensers, connected three in parallel, three in series, (0.01 microfarad) were next tried. The oscillogram, Fig. 11, shows the decided improvement. The attempt to produce further improvement in the voltage by this means was abandoned as it was evident that a far larger amount of capacity than was available would be required to obtain a voltage varying less than one per cent.

As the impedance of the load (the voltmeter and corona tube) was high, the current which flowed was very small. Under these conditions a moderate amount of series inductance (50 or 100 henrys) in conjunction with capacity offered a probable means of further improvement if connected in the manner shown in Fig. 3, where the capacity is divided into two portions. The voltage fluctuation at the terminals of C_1 is an irregular shaped wave which consists of a fundamental with a frequency of 600 cycles per second and a number of higher harmonics. The inductance L_2 and the capacity C_2 offer a definite value of impedance to each of these higher frequencies. The higher the frequency the greater the impedance offered by the inductance, L_2 or in other words the voltage wave is smoothed out by removing the higher harmonics. The current drawn by the condenser, C_2 over the inductance L_2 stores up electromagnetic energy in the latter. When the voltage of the condenser starts to fall, due to the current drawn from it by the load, the energy stored up in the inductance is delivered to the circuit and tends to maintain the voltage at the terminals of C_2 constant.

A number of inductances were tried in the position L_2 and the sound in a telephone used as a measure of the degree of voltage fluctuation. The intensity of sound in the telephone is proportional to the amplitude of the voltage fluctuation. The minimum sound in the telephone was obtained from one half of the high-tension winding of a 5-kw., 6600/110-volt power transformer. This final arrangement was used throughout the tests.

It is possible to calculate the voltage fluctuation under test conditions, if it is assumed that is directly proportional to the current. Referring to Fig. 3, the values were as follows, when the oscillograph was in circuit:

$$C_1 = \frac{3\,(0.01)}{2} = 0.015 \text{ microfarad}$$

$$C_2 = \frac{2\,(0.01)}{2} + 0.032 = 0.042 \text{ microfarad}$$

$$C = C_1 + C_2 = 0.015 + 0.042 = 0.057 \text{ microfarad}$$

$$= 5.7 \times 10^{-8} \text{ farads}$$

V = direct-current voltage = 4700

δV = voltage fluctuation = 13.8 per cent = 650 volts. (By measurement on the oscillogram)

i_1 = oscillograph current = 0.024 ampere (By measurement)

i_2 = voltmeter current = 0.001 ampere (By calculation)

i = total current flowing when oscillograph is connected.

$$= i_1 + i_2 = 0.025 \text{ ampere}$$

$$\omega = 2\pi \times \text{frequency} = 2\pi \times 600 = 1200\pi$$

Hull[7] has shown that the voltage fluctuation (δV) at the terminals of C_2 may be expressed by the simple relation

$$V = \frac{8\,i\,\pi}{L_2\,\omega^3}\left(C + \frac{1}{L_2\,\omega^2}\right)^2 \qquad (2)$$

where i, ω, and C are as given above and L_2 is the magnitude of the series inductance in henrys. If we assume L_2 constant, (*i.e.*, independent of the amount of current due to the low flux density in the iron of the transformer which was used as L_2), we can write

$$\delta V = K\,i \qquad (3)$$

Let δV_a = voltage fluctuation when current i is flowing.
δV_b = voltage fluctuation when current i_2 is flowing
Then, $\delta V_a = K\,i$; and, $\delta V_b = K i_2$. (4) and (5)

Dividing (**4**) by (**5**) and transposing,

$$\delta V_b = \frac{i_2}{i} \times V_a \qquad (6)$$

$$= \left(\frac{0.001}{0.025} \times 13.8\right) \text{ per cent} = 0.5 \text{ per cent}$$

= percentage voltage fluctuation under test conditions, (*i.e.*, voltmeter and corona tube.)

A comparison of the different methods of detecting corona was then made. In the case of the alternating-current corona the

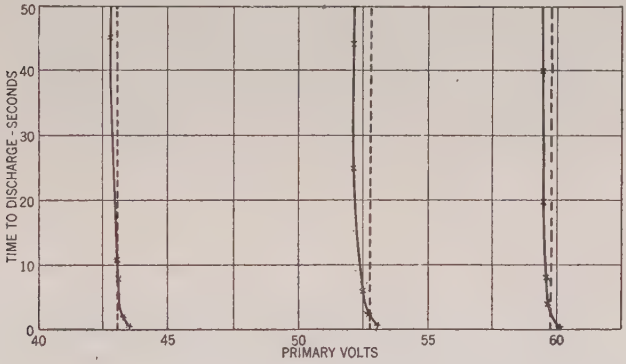


FIG. 13—VISUAL CORONA AND ELECTROSCOPE DISCHARGE
– – – Visual corona
—— Electroscope discharge
Diameter tube = 6.109 cm.
Diameter wires = 0.107—0.166—0.231 cm.

visual and electroscope methods were compared. The curves (Fig. 13) show how closely the two methods agree, when the bend in the electroscope curve is taken as the point at which corona begins. In the case of the direct-current corona an additional method was available, *i.e.*, a galvanometer connected between the tube and ground as used by Farwell and Mackenzie[8]. A ballistic galvanometer with a sensitivity of 10^{-6} amperes per mm. scale division when the scale was placed one meter from the mirror, was used. Below corona voltage if the wire was clean, the galvanometer stood practically on zero. With the appearance of corona, the galvanometer took a large deflection (5 to 10 cm.) and any further increase in voltage threw the light spot off the scale.

The condition of the surface of the wire when fairly clean had but little effect on the value of the critical surface intensity in the case of the wire at positive or alternating-current potentials, but with the wire negative the slightest surface imperfection or particle of dust caused a lowering of the value of E. The observations on negative corona were extremely difficult on this account. On raising the voltage the slightest amount above that necessary to form corona, the blue glow changed to a purple point discharge and on lowering the voltage, corona held on to a value of voltage far below that required to start corona. The wire had to be removed and carefully polished again before another observation could be made. With precautions against the above increase of voltage beyond corona value it was possible to repeat the observations on the negative corona as often as desired.

Final Methods of Taking Observations. The wire was carefully polished, placed in the tube, accurately centered and connected to the voltage supply. In the alternating-current case the voltage was slowly raised by increasing the generator field current until the electroscope began to discharge. Readings were taken of the rate of leak of the electroscope corresponding to each increment of the voltage, until the rate of discharge became practically infinite. The field current was then decreased until the leak of the electroscope ceased and then increased again until visual corona appeared. The voltage was read at this instant on the primary voltmeter and recorded as primary volts. The temperature and pressure were also recorded. The peak factor of alternating voltage waves was determined by means of oscillograms. A large number were taken and from these the values of 1.46 at 40 volts, 1.46 at 50 volts and 1.46 at 60 volts were determined by measurement. In order to give an idea of the conditions of accuracy in making these measurements, the figures in Table III are given for one half of the wave of the oscillogram corresponding to 60 volts. The curve was carefully traced on co-ordinate paper and 26 ordinates, 1/20 of an inch (1.27 mm.) apart were read off; the height of each ordinate could be read to 1/50 of an inch (0.508 mm.) The maximum ordinate was 1.43 inches (37.08 mm.) high and the r.m.s of all the ordinates was 0.977, which gives a ratio of maximum to effective of 1.464. Similar measurements on the other oscillograms gave the values as stated above. The average of these three (1.46) was taken as the ratio of maximum to effective value over the range 40-60 primary volts.

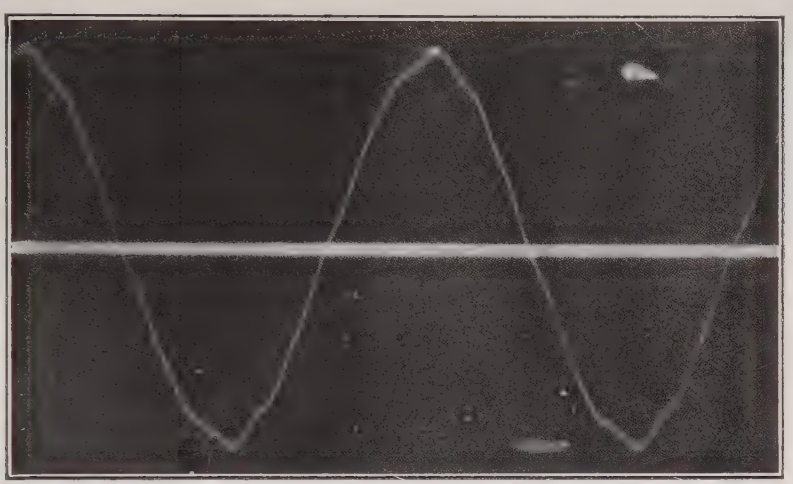
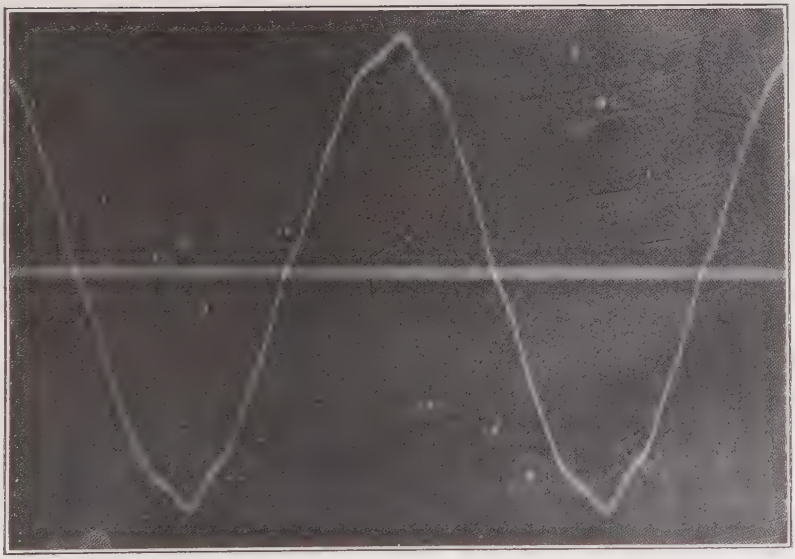


FIG. 14—AT 40 VOLTS



[WHITEHEAD AND BROWN]

FIG. 15—AT 60 VOLTS

TABLE III—OSCILLOGRAM OF THE VOLTAGE ACROSS THE LOW-TENSION TERMINALS OF THE TRANSFORMERS AT 60 VOLTS

Ordinate No.	Length	(Length)
1	0	0
2	2.5	6.25
3	5.0	25.00
4	7.0	49.00
5	11.5	132 25
6	15.3	234.09
7	17.8	316.84
8	20.3	412.09
9	22.3	497.29
10	24.2	585.64
11	25.0	625.00
12	26.0	676.00
13	27.0	729.00
14	28.0	784.00
15	28.6	817.96
16	27.0	729.00
17	25.2	635.04
18	23.5	552.25
19	22.7	515.29
20	21.0	441.00
21	17.8	316.84
22	15.0	225.00
23	12.0	144.00
24	8.8	77.44
25	5.2	27.04
26	0.0	0.00

The oscillograms taken at 40 and 60 volts respectively are given in Fig. 14 and 15.

In the direct-current experiments the procedure was much the same, the high-tension voltage however being read directly on the voltmeter in series with the multiplier, no peak factor determination being necessary. In the case of the negative corona the electroscope curve could not be obtained due to the disadvantage of raising the voltage above corona as noted above and finally, the galvanometer method of detecting the corona was used.

Accuracy of Observations. The accuracy of observation in the case of the alternating-current corona was limited only by the accuracy of reading the voltmeter, as the observations could be repeated again and again from day to day when correction was made for temperature and pressure, and the other factors entering into the determination of the value of E could be determined to a small fraction of 1 per cent. The voltmeter could be read to 1/5 of a division at 60 volts or about 1/3 per cent. It is

recognized, however, that the accuracy also depended upon the measurements of peak factor on the oscillograms. The method of measuring these has already been outlined and as the value of the peak factor varied only 0.07 per cent over the range of voltage used, it is felt that the measurement of voltage is the limiting factor, as stated above. The accuracy in the continuous current case was also limited by the voltmeter reading and was about 1/3 per cent also.

IV. Final Experiments

Relation Between Critical Surface Intensity and Diameter of the Wire for Alternating Voltage. In Table IV are given the results of observations on six sizes of wire in a tube 6.109 cm. in diameter. The first column gives the wire diameter in centimeters, the second column the "density factor" δ, as obtained from Peek's relation

$$\delta = \frac{3.92\,\rho}{273 + t} \tag{7}$$

and the third column the primary volts. V the critical voltage (maximum) is obtained by multiplying the primary voltage by the ratio of transformation and the peak factor. The relation

$$E = \frac{V}{a \log_e b/a} \tag{8}$$

was used in calculating the critical surface intensity E.

Relation Between Critical Surface Intensity and Diameter of the Wire for Positive and Negative Continuous Voltage. In Table V are given the results of observations on the same wires for positive corona and in Table VI for negative corona. The first column is again the wire diameter, the second column the density factor calculated from the observed temperature and pressure and the third column the voltmeter reading. The critical voltage V is obtained by multiplying the voltmeter reading by the multiplying factor. The last column is obtained as before.

The small number of observations on the negative corona is due to the great difficulty experienced in obtaining them as mentioned before. As many as ten trials with their attendant polishing, centering, etc., were often required to obtain a stable corona. The galvanometer in the grounded circuit proved a great help as the galvanometer only remained on zero up to

corona forming voltage when the wire was absolutely clean and if at no time during the run the voltage had been raised above that required to form corona on the given wire.

Determination of the Constants A and B of Formula (**1**). As stated in the introduction the formula connecting E (the critical surface intensity) with the diameter of the wire is of the form

$$E = A\,\delta + B\sqrt{\frac{\delta}{d}} \qquad (1)$$

TABLE IV—ALTERNATING CURRENT

Diam. wire cms.	Density factor δ	Primary volts	Crit. voltage (maximum) V	Crit. surface intensity E (kv.)
0.074	1.005	35.8	13,120	80.6
	1.005	35.8	13,120	80.6
	1.005	35.8	13,120	80.6
0.090	1.005	39.65	14,530	77.1
	1.005	39.65	14,530	77.1
	1.005	39.65	14,530	77.1
0.107	1.005	42.85	15,700	72.6
	1.005	43.05	15,780	73.0
	1.005	43.20	15,830	73.2
0.125	1.005	45.6	16,700	68.8
	1.005	45.65	16,720	68.9
	1.005	45.7	16,750	69.0
0.166	1.005	53.5	19,610	65.6
	1.005	53.5	19,610	65.6
	1.005	53.5	19,610	65.6
0.231	1.005	61.4	22,500	59.6
	1.005	61.4	22,500	59.6
	1.005	61.4	22,500	59.6
0.231	1.000	64.2	23,520	59.7
	1.000	64.2	23,520	59.7
	1.000	64.2	23,520	59.7

where, E = critical surface intensity in kilovolts per centimeter.

δ = Density factor.

A and B = Constants.

To determine these constants from the observed results the method of least squares was used. The calculation as applied to the positive corona is given in full in Table VII. Attention is

TABLE V—POSITIVE CORONA

Diam. wire cms.	Density factor δ	D-C. volts	Mul. fac.	Critical voltage (V)	Crit. surf. intensity (E)
0.074	1.02	228.0	52.16	11,897	77.0
	1.02	227.5	52.18	11,871	76.9
	1.02	228.0	52.18	11,897	77.0
	1.00	558.0	19.83	11,660	75.5
	1.00	587.5	19.83	11,650	75.4
	1.00	587.5	19.83	11,650	75.4
	1.022	227.5	52.18	11,871	77.1
	1.022	227.5	52.18	11,871	77.1
	1.022	227.5	52.18	11,871	77.1
	1.035	442.5	28.36	12,549	77.1
	1.035	443.0	28.36	12,560	77.2
	1.035	443.0	28.36	12,560	77.2
0.090	1.01	657.5	19.83	13,038	72.8
	1.01	657.5	19.83	13,038	72.8
	1.01	657.5	19.83	13,038	72.8
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.007	637.5	19.83	13,038	72.8
	1.007	367.5	19.83	13,038	72.8
	1.007	637.5	19.83	13,038	72.8
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.02	665.0	19.83	13,187	73.6
	1.005	657.5	19.83	13,038	72.8
	1.005	657.5	19.83	13,038	72.8
	1.005	657.5	19.83	13,038	72.8
	0.998	654.0	19.83	12,969	71.4
	0.998	654.0	19.83	12,969	71.4
	0.998	654.0	19.83	12,969	71.4
	1.025	253.0	52.18	13,200	73.7
	1.025	253.0	52.18	13,200	73.7
	1.025	253.0	52.18	13,200	73.7
0.107	1.022	275.0	52.18	14,350	69.2
	1.022	275.0	52.18	14,350	69.2
	1.022	275.0	52.18	14,350	69.2
	1.035	533.0	28.36	15,116	69.9
	1.053	532.5	28.36	15,102	69.8
	1.035	533.0	28.36	15,116	69.9
0.125	1.008	292.5	52.18	15,263	66.8
	1.008	292.5	52.18	15,263	66.8
	1.008	292.5	52.18	15,263	66.8
0.166	1.003	357.5	52.18	18,654	62.4
	1.003	357.5	52.18	18,654	62.4
	1.003	357.5	52.18	18,654	62.4
0.231	1.02	422.5	52.18	22,046	58.4
	1.02	422.5	52.18	22,046	58.4
	1.02	422.5	52.18	22,046	58.4

called to the excellent agreement in the observed results and those calculated by the formula. The observed values are given in the ninth column of Table VII, the calculated values in next to the last column and the percentage error referred to the observed values in the last column. This error is less than 1 per cent with the exception of the wire of radius 0.0535 cms.

Similar calculations were made for the alternating voltage and

TABLE VI—NEGATIVE CORONA

Diam. wire cms.	Density factor δ	D-C. volts	Mul. fac	Critical voltage (V)	Crit. surf. intensity (E)
0.074	1.003	462.5	28.36	13,117	80 6
	1.003	462.5	28.36	13,117	80.6
	1.003	462.5	28.36	13,117	80.6
0.090	1.03	517.0	28.36	14,662	77.8
	1.03	517.5	28.36	14,676	77.9
	1.03	517.0	28.36	14,662	77.8
	0.9907	503.0	28.36	14,265	75.6
	0.9907	502.5	28.36	14,230	75.6
	0.9907	503.0	28.36	14,265	75.6
0.107	1.003	560.0	28.36	15,880	73.5
	1.003	560.0	28.36	15,880	73.5
	1.003	560.0	28.36	15,880	73.5
1.125	0.992	587.0	28.36	16,657	68.6
	0.992	587.0	28.36	16,657	68.6
	0.992	587.0	28.36	16,657	68.6
0.166	1.003	667.5	28.36	19,214	64.3
	1.003	667.5	28.36	19,214	64.3
	1.003	667.5	28.36	19,214	64.3
0.231	1.001	431.0	52.18	22,440	59.3
	1.001	431.0	52.18	22,440	59.3
	1.001	431.0	52.18	22,440	59.3

the negative continuous voltage and the following constants were found.

Positive Corona

$A = 33.7$ $B = 11.5$

Negative Corona

$A = 31.02$ $B = 13.5$

Alternating-Current Corona

$A = 33.7$ $B = 12.6$

TABLE VII—CALCULATION OF CONSTANTS A AND B USING METHOD OF LEAST SQUARES. POSITIVE CORONA

r	δ	δ^2	δ^3	$\frac{\delta^3}{r}$	$\sqrt{\frac{\delta^3}{r}}$	$\frac{\delta}{r}$
0.037	1.02	1.0404	1.061	28.68	5.375	27.77
0.045	1.025	1.0506	1.076	23.9	4.885	22.7
0.0535	1.022	1.0445	1.068	19.97	4.47	19.14
0.0625	1.008	1.0161	1.025	16.4	4.05	16.12
0.083	1.003	1.0060	1.01	12.16	8.85	12.07
0.1155	1.02	1.0404	1.061	9.19	3.03	8.82
Σ		6.198			25.235	106.62

r	E obs.	δE	$\sqrt{\frac{\delta}{r}}$	$\sqrt{\frac{\delta}{r}} E$	E Calc.	Error % per cent
0.037	77.0	78.54	5.27	405.79	77.1	− 0.13
0.045	73.7	75.54	4.765	351.18	73.4	+ 0.41
0.0535	69.2	70.72	4.375	302.75	70.0	− 1.1
0.0625	66.8	67.33	4.015	268.2	66.7	+ 0.14
0.083	62.4	62.59	3.475	216.84	62.1	+ 0.48
0.1155	58.4	59.56	2.97	173.45	58.5	− 0.17
Σ		414.28	24.87	1718.21		

Resultant values $A = 437.7$, $B = 11.5$.

The values of E for the six sizes of wire calculated from the formula,

$$E = A + \frac{B}{\sqrt{d}}$$

using the above constants are given in Tables VIII and IX and plotted in Fig. 16.

TABLE VIII—ALTERNATING CURRENT

RELATION BETWEEN DIAMETER AND CRITICAL SURFACE INTENSITY

Diam. wire cms.	Density factor δ	Primary volts	Critical voltage (max.) (V)	Crit. surf. intensity E (kv.)/cm.
0.074	1.005	35.8	13,120	80.6
0.090	1.005	39.65	14,530	77.1
0.107	1.005	43.0	15,760	72.9
0.125	1.005	45.7	16,750	69.0
0.166	1.005	53.5	19,610	65.6
0.231	1.005	61.4	22,500	59.6

TABLE IX—CONTINUOUS CURRENT

(Positive)

Relation between Diameter and Critical Surface Intensity

Diam. wire cms.	Density factor δ	Voltmeter reading	Critical voltage (V)	Crit. surf. intensity. E (kv.)/cm.
0.074	1.02	228.0	11,897	77.0
0.090	1.025	253.0	13,200	73.7
0.107	1.022	275.0	14,350	69.2
0.125	1.008	292.5	15,263	66.8
0.166	1.003	357.5	18,654	62.4
0.231	1.02	422.5	22,044	58.4

(Negative)

Diam. wire cms.	Density factor δ	Voltmeter reading	Critical voltage (V)	Crit. surf. intensity E (kv.)/cm.
0.074	1.003	462.5	13,117	80.6
0.090	1.03	517.0	14,662	77.8
0.107	1.003	560.0	15,880	73.5
0.125	0.992	587.0	16,657	68.6
0.166	1.003	677.5	19,214	64.3
0.231	1.001	431.0	22,490	59.3

In Table X is given a comparison of the results of observations on positive, negative and alternating-current corona with the results of the more important earlier researches. The formula,

$$E = A\,\delta + B\sqrt{\frac{\delta}{d'}}$$

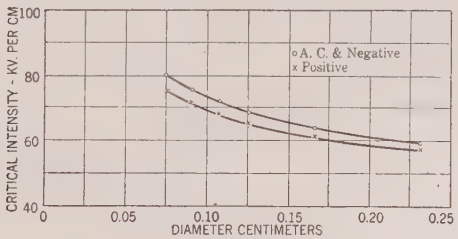


Fig. 16—Corona Voltages for Round Conductors

was used in all cases and the values of the constants A and B (as taken from the accounts of the different investigations and given in Table XI), substituted in order to obtain the values of E given in Table X.

The values of Table X for the alternating-current case are plotted in Fig. 17, those for the continuous current case (positive and negative) in Fig. 18.

TABLE X—COMPARISON OF RESULTS

CRITICAL SURFACE INTENSITY: KILOVOLTS PER CENTIMETER

Alternating current				Continuous current			
Diam. wire cm.	Whitehead	Peek	Whitehead & Brown	Whitehead & Brown +	Farwell +	Whitehead & Brown −	Farwell −
0.074	81.2	80.7	80.3	76.0	75.7	80.8	76.9
0.090	76.6	76.0	76.0	71.9	71.6	76.1	72.0
0.107	72.9	72.3	72.5	68.9	68.3	72.4	69.8
0.125	69.9	69.2	69.6	66.2	65.5	69.3	67.2
0.166	64.8	64.1	64.8	61.9	61.1	64.2	62.9
0.231	59.9	59.1	60.1	57.6	56.6	59.1	58.7

DISCUSSION

The fact that positive corona appears at a lower voltage than negative for the same size wire is in accord with the theory of secondary ionization if we assume that the negative ion is the principal ionizing agent. This is practically certain owing to

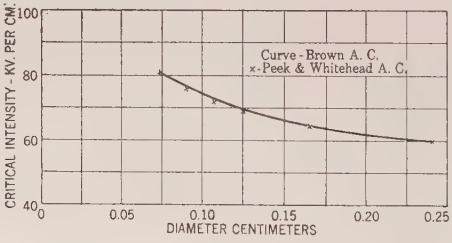


FIG. 17—CORONA VOLTAGES FOR ROUND CONDUCTORS

its energy of motion being greater than that of the positive ion under a given voltage gradient. If a difference of potential be applied between a wire and concentric cylinder the negative ions or electrons present in the air will move under the influence of the electric field. In so doing they will collide with the molecules of the air and they may or not produce other ions, depend-

ing upon their velocity. Now suppose the wire is positive. In this case the electrons will move inwards (toward the wire) and in so doing will move in the direction in which the intensity of the field is increasing. Thus new ions caused by collisions are formed in stronger and stronger fields and there is a consequent tendency to multiply to the concentration corresponding to corona formation. On the other hand if the wire is negative,

TABLE XI

Type of voltage	Name of investigator	Constants	
		A	*B*
a-c.	Whitehead	32	13.4
a-c.	Peek	31	13.5
d-c.	Farwell	31.6	11.9
d-c.	Farwell	35.0	11.4

the electrons will move outwards in the direction of decreasing field intensity. Only those which start from near the wire pass through the field of strong electric force so that the number of ions produced by collisions is much smaller than in the first case or a higher difference of potential will be required to produce enough ions by collision to start corona.

There is nothing new in the convergence of the curves for

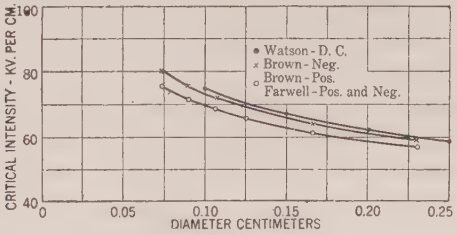


FIG. 18—CORONA VOLTAGES FOR ROUND CONDUCTORS

positive and negative corona. Schaffers, Watson, Mackenzie and Farwell all found evidences of this. As the meeting point is a function of the pressure and the diameter of the wire it is hard to compare the results of the different investigations on this basis. If the curves for positive and negative corona be extended they will meet at a point corresponding to a wire of diameter 0.285 cms.

Farwell states that the appearance of negative corona changes with increase of wire diameter and voltage, that the negative corona started with a bright spot or two on small wires followed by a continuous brush discharge as the voltage was increased and that for the larger wires the brush discharge appeared immediately upon reaching the critical voltage. In the present experiments by careful polishing and by raising the voltage in very small steps it was found possible to make the negative corona appear sharply as a continuous bluish glow on all the wires used. If the voltage was raised the smallest amount above that necessary to produce corona, the bluish glow collapsed to a purplish, point discharge and by lowering the voltage until the discharge just disappeared, a value of E was obtained in close agreement with Farwell. It is believed that this explains to some extent the low values obtained by Farwell in this case.

V. Experiments Testing the Theory of Corona Formation

Theory of the Start of Corona. According to the theory of secondary ionization, if a difference of potential be applied to a tube and concentric wire, the free electrons present in the air between will acquire a velocity under the electric field. The velocity will depend on the applied difference of potential and the "mean free path" of the electron. In order that corona form about the wire it is necessary that the electron acquire a velocity sufficient to produce other electrons by collision and that this process multiply to some state of concentration constituting corona. If the possible free path of the electron be limited in any way, a higher difference of potential will be required to give the critical velocity to the electron or to produce corona.

To test this theory concentric metal tubes with thin walls and insulating supports were introduced between the wire and the corona tube proper, in order to reduce the possible free path of the negative ion under the influence of the electric field and to observe the effect of this reduction on the value of V, the maximum value of the voltage required to produce corona. The great difficulty in such experiments is to determine when corona starts when the secondary tube is in place. On applying continuous voltage to the wire it was found that the voltage could be carried far above that required to start corona when the auxiliary tube was not in place without the slightest deflection of a galvanometer connected between the tube and ground and

without any indication of glow about the wire. It was at first thought that this could be explained on the hypothesis that the intermediate tube cut down the number of ionizing electrons already in motion in the outer portion of the field in accordance with the indications of the theory. An electrostatic voltmeter was then connected between the inner tube and the ground and the voltage again applied. On removing the voltage from the central wire the intermediate tube was found to retain a potential, as shown by the electrostatic voltmeter. An electroscope carrying a charge of known sign showed that the intermediate tube was charged to a potential of the same sign as the wire.

The failure of the galvanometer to deflect or of corona to appear was now easily explained; the presence of the high potential of the same sign on the intermediate tube lowered the potential gradient between the wire and the outer tube and therefore prevented the appearance of corona. That this is the correct explanation was shown by the absence of charge on the inner tube for voltages below corona. The conclusion therefore is that the corona may form at normal voltage, generating positive and negative ions. The ions of the opposite sign are attracted to the wire, those of like sign are driven to the intermediate tube where they raise its potential. This elevation of potential continues until it is sufficient to lower the intensity at the surface of the corona wire below corona forming value. The whole process is probably instantaneous.

Alternating current was then applied to the wire as it was thought that the inner tube would not acquire the charge noted above. The kenotron was used to rectify the current which flowed from the outer tube to the ground through the galvanometer, which was retained in these alternating-current experiments by reason of its value as a detector. Here again trouble was encountered due to a charge accumulating on the outer tube which disturbed the potential gradient and made it impossible to obtain corona even on raising the voltage far above that required to start corona when the outer tube alone was used and connected directly to ground.

The kenotron is a perfect rectifier and allows current to flow in one direction only. During the half cycle when the wire is positive a charge of the opposite sign is acquired by the tube through the kenotron. During the next half cycle the wire becomes negative, but a positive charge does not accumulate on the outer tube as the kenotron is conducting in but one direction.

The negative charge already there cannot return through the kenotron for the same reason and with each successive half-cycle more and more charge accumulates until the outer tube is considerably above ground potential, which lowers the effective potential gradient and prevents the formation of corona.

TABLE XII

INFLUENCE OF CONCENTRIC SCREENING TUBES ON CORONA VOLTAGE

(WITHOUT INNER TUBE)

A-C. volts	Shunt in ohms	$\frac{r_1 + r_2}{r_1}$
40.0	11.0	3.780
45.0	10.0	4.060
50.0	9.0	4.400
55.0	8.5	4.605
60.0	7.5	5.080
65.0	7.0	5.340
67.5	6.0	6.100
70.0	5.0	7.120
72.5	3.0	11.200

(WITHOUT INNER TUBE)

A-C. volts	Shunt in ohms	$\frac{r_1 + r_2}{r_1}$
40.0	12.0	3.56
45.0	11.0	3.78
50.0	10.0	4.06
55.0	9.3	4.30
60.0	8.7	4.52
62.5	8.3	4.69
65.0	7.6	5.03
66.0	6.5	5.71

Galvanometer Deflection Constant = 1 cm.
Visual Corona With Tube = 69.1
Visual Corona Without Tube = 64.2

A vibration galvanometer in the ground connection was next used and after many trials the results shown in Table XII and plotted in Fig. 19 were obtained. The instrument used was of the soft iron magnet type, in which a piece of soft iron is suspended by a silk fibre between the poles of a horseshoe magnet. A coil carrying the alternating current to be measured produces a field perpendicular to that of the magnet. The temporarily

magnetized piece of iron vibrates in synchronism with the alternating current and with an amplitude proportional to the strength of the current. The period of the moving system is adjusted to resonance by a magnetic shunt on the limbs of the magnet.

The first column gives the primary volts. The deflection of the galvanometer was held constant at one centimeter, by varying a non-inductive shunt across the instrument. The values of the shunt used are given in the second column. The total current flowing from the tube at any given voltage on the wire, is given by

$$i = \frac{r_1 + r_2}{r_1} + i_1 \qquad (9)$$

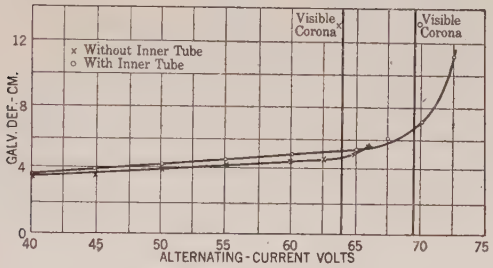


FIG. 19—EFFECT OF CONCENTRIC TUBES
Diameter outer cube = 6.98 cm.
Diameter inner tube = 2.448 cm.
Diameter wire = 0.231 cm.

where, r_1 = resistance of the galvanometer.
r_2 = resistance of the shunt.
i_1 = current through the galvanometer.

As the galvanometer current was held constant, the above equation reduces to

$$i = k\frac{r_1 + r_2}{r_1} \qquad (10)$$

The values of $\frac{r_1 + r_2}{r_1}$ are entered in the last column and the curve (Fig. 19) is plotted between $\frac{r_1+r_2}{r_1}$, which is proportional to the total current flowing, and the primary volts, on the transformer. The values at which visual corona appeared with and without this tube, which was 2.448 cms. in diameter, are given

in Table XIII and also marked on the curve. Three other tubes were inserted and observations made on the visual corona with and without them. These values are also entered in the table.

TABLE XIII

EFFECT OF THIN CONCENTRIC INSULATED TUBES ON CORONA VOLTAGE.

Diameter inner tube cms.	Primary volts
2.448	68.7
1.829	69.1
1.209	62.0 (Spark over)
0.605	.... (" ")
No tube	64.2
Diameter outer tube 6.98 cm.	Diameter wire 0.231 cm.

Discussion. The curve (Fig. 18) shows that the charging current is higher when the intermediate tube is in place as it should be owing to the increased capacity. The curves follow straight lines up to a value of voltage required to produce corona with no intermediate tube. An indication is given that the corona current rises more rapidly when the tube is not in place which is in accord with the theory. Corona appears at a higher value of voltage when either of the larger tubes is inserted which is in accord with the theory, as they cut down the number of ionizing electrons already in motion due to the outer portion of the field. If we omit the value obtained for the 1.209 cm. tube which is uncertain owing to spark over, we see that the smaller the diameter of the inner tube the higher the voltage required which is again in accord with the theory as the smaller the inner tube the more the accelerating path is cut down and the higher the voltage required to start corona. Because of the difficulty of maintaining the galvanometer deflection constant (it is affected by the slightest variation in the frequency) further modifications in the direction of constancy of frequency are necessary, and the experiments were terminated at this point. It is believed however that the above results are a reliable qualitative indication that corona voltage is elevated by an intermediate thin screen which serves as a barrier to the free passage of the ionizing electrons.

VI. CONCLUSIONS

1. The critical corona forming intensity at atmospheric pressure has been determined for six sizes of wire ranging from 0.074

to 0.231 cms. diameter for alternating and continuous potentials, in the same apparatus and under the same conditions.

2. The relation between critical surface intensity, that is, the intensity at which corona starts, and the diameter of a clean round conductor as found are expressed by the following laws:

$$\text{Alternating current}^* \quad E = 33.7\,\delta + 12.6\sqrt{\frac{\delta}{d}}$$

$$\frac{\text{Continuous current}^\dagger}{\text{Positive}} \quad E = 33.7\,\delta + 11.5\sqrt{\frac{\delta}{d}}$$

$$\frac{\text{Continuous current}^\ddagger}{\text{Negative}} \quad E = 31.02\,\delta + 13.5\sqrt{\frac{\delta}{d}}$$

3. Within the range of wires used the corona appears at a lower voltage when the wire is positive than when it is negative. The maximum excess of negative over positive observed was 6.3 per cent.

4. The values with alternating voltage coincide closely with those of negative continuous voltage.

5. The curves representing the positive and negative values converge as the diameter of the wire increases.

6. The observations on the positive and alternating-current corona are in substantial agreement with those of Farwell, Peek and Whitehead, although new values of the constants of formula (**1**) are indicated.

7. The observations on the negative corona give values higher than any heretofore obtained.

8. Experiments with concentric insulated screening tubes indicate that the critical corona forming intensity is raised by their introduction. (See Table XIII and Fig. 18.)

9. The results of the present investigation are in accord with the theory of secondary ionization as proposed by Townsend.

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*(See Table VIII and Fig. 15.)

†(See Table IX and Fig. 15.)

‡(See Table IX and Fig. 15.)

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